Variable-Structure Control of Spacecraft Attitude Maneuvers

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A variable-structure control approach is presented for multiaxial spacecraft attitude maneuvers. Nonlinear sliding surfaces are proposed that result in asymptotically stable, ideal linear decoupled sliding motions of Cayley-Rodrigues attitude parameters, as well as of angular velocities. The resulting control laws are interpreted as more easily implemented and more robust versions of those previously obtained by feedback linearization.

I. Introduction

MULTIAXIAL large-angle spacecraft attitude maneuvers generally pose nonlinear dynamic control problems, which defy on-line solution with limited onboard computational capabilities. A number of approaches have been proposed for the adequate treatment of such problems. Linearization around nominal points or else a sequence of single-axis maneuvers, as in Breakwell[1] and Hefner et al.[2] eliminate the nonlinear nature of the problem, but at the expense of slow response and underutilization of actuators. Nonlinear optimal control theory has also been applied to this class of problems, such as by Junkins and Turner.[3] An immediate advantage of that approach is the possibility of considering multiaxial rotations without resorting to a single-axis decomposition strategy. The computational burden is, however, significantly increased, and off-line two-point boundary-value problems (TPBVP) have to be solved for each required maneuver. Other works, such as by Vadali and Junkins,[4] address the same problem by using the method of particular solutions of the intrinsic TPBVP. Other direct solution methods involve a combination of optimal control theory and polynomial feedback control approximation such as by Dwyer[5] and by Carrington and Junkins.[6] Recently, exact feedback linearization, as in Hunt et al.[7] has found application to spacecraft attitude maneuvers, such as by Dwyer[8,10] and by Dwyer and Batten.[9] In that approach, nonlinear slewing problems are solved by formulating maneuvers based on an equivalent Brunovsky canonical version of the system, obtained through nonlinear transformation of coordinates and nonlinear feedback. The effect of elastic deformations can also be taken into account, as shown by Dwyer[11] and by Monaco and Stornelli,[12] requiring, however, elastic deformation feedback for torque profile correction.

In the recent paper by Vadali,[13] the use of variable-structure control (VSC) for large-angle rotational maneuvers has been proposed, especially when pulsed-width, pulsed-frequency modulation thrusters are available (as discussed by Wie and Barba[14]). An elegant optimal control approach for the sliding surface synthesis problem was formulated by Vadali,[13] through minimization of mean-square quaternion error and mean-square angular velocity, to yield a solution for the sliding surface in closed form. The sliding surface relating attitude quaternion and angular velocity was shown to be linear for each axis. That approach permits a simple controller design that is also robust with respect to disturbances. However, the stability characteristics of the ideal sliding kinematics could not be independently prescribed for each orientation parameter, although asymptotic stability for all axes was achieved, with a common preselected exponential rate of decay for a function of the Euler attitude parameters.

Motivated by the work of Vadali,[13] in this paper a more general VSC scheme is proposed, based on nonlinear sliding manifolds,[15–17] defined either in the spacecraft kinematic or dynamic variables. Sliding regimes are found that result in a controlled reduced system on which the relaxation time of each attitude or rate coordinate can be independently chosen. As with Vadali,[13] the resulting control laws require only an estimate of the computed torques, as well as a switching logic entirely determined by the desired kinematics, which are now freely chosen. In contrast, earlier work[8–11] relied on the exact computation of the required torque profiles, with attendant complexity and model sensitivity both alleviated by the now proposed "overshoot and switch" scheme. Inasmuch as the effects of elastic distortion on line-of-sight motion can be accommodated by increasing VSC gains, only rigid motion will be modeled here.

In Sec. II of this paper, nonlinear equations of motion are discussed for the control of a spacecraft undergoing multiaxial rotational maneuvers. Following Wang,[18] Cayley-Rodrigues attitude parameters are used instead of Euler angles or quaternions, for rationality, nonredundancy, and nonsingularity. Following Utkin,[19] sliding surfaces and control effort estimates are then obtained in Sec. III, either for the attitude or the rate variables. The use of interpolation[20] to avoid chattering is also proposed here for the synthesis of the actual commanded torque generation; i.e., saturating torque actuators can be assumed rather than ideal torque switchings. This is done at the expense of acceptance of designer-selected attitude error margins. Section IV contains applications to reorientation and Sec. V to detumbling maneuvers. Section VI contains the conclusions and discussion.

II. Equations of Motion

Kinematics and Dynamics

Given a preselected inertial reference frame, a spacecraft equipped with pairs of opposing thrusters, or else with reaction wheels with axes aligned along the principal axes of inertia, is governed by the following idealized equations of motion in the body axes:

$$\frac{d}{dt}\xi = T(\xi)\omega := \tfrac{1}{2}[I + \xi\xi^T + \xi\times]\omega \tag{1}$$

$$J\frac{d}{dt}\omega = h(\xi)\times\omega + \mu \tag{2}$$

with its terms defined as follows: ω denotes the angular velocity vector, resolved along the principal axes; ξ is the Gibbs vector of Cayley-Rodrigues attitude parameters defined as follows:

$$\xi = e\tan(\phi/2) \tag{3}$$

denoting the result of a virtual rotation by ϕ rad about a virtual unit axis vector e (Euler axis), with the same components

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e_1, e_2, e_3, along either the preselected inertial reference axes or the body axes, so that $e^T e = 1$; μ symbolizes either externally applied thruster torques τ^0, i.e., $\mu = \tau^0$, or the negative of the torques $\tau' = \mathrm{col}(\tau_1', \tau_2', \tau_3')$ driving the reaction wheels, i.e., $\mu = -\tau'$; I is the 3×3 identity matrix; J stands either for the system's inertia matrix I^0, i.e., $J = I^0$, or that inertia with the diagonal matrix I' of axial wheel moments subtracted, i.e., $J = I^0 - I'$; and h is the system's angular momentum in body coordinates.

For externally applied torques with locked wheels, or else with no thruster torques but active wheels or control moment gyros, one has the following representations for the angular momentum:

$$h = \begin{cases} I^0 \omega & (\tau' = 0) \\ A(\xi)A[-\xi(0)]h(0) & (\tau^0 = 0) \end{cases} \tag{4}$$

where $A(\xi)$ denotes the change of variables matrix from the inertial axes to the body axes, which is parametrized as follows[21,22]:

$$A(\xi) = 2(1 + \xi^T \xi)^{-1}[I + \xi\xi^T + \xi \times]\omega \tag{5}$$

The inverse of $A(\xi)$ is $A(-\xi)$. The inverse of the kinematic equation is likewise globally defined:

$$T(\xi)^{-1} = 2(1 + \xi^T \xi)^{-1}[I - \xi \times] \tag{6}$$

The second form of Eq. (4) for the angular momentum arises from h being inertially constant in the absence of external torques. The modeling of structural deformation effects introduces additional terms in Eqs. (2) and (4), as well as the need also to add the structural dynamics to the equations of motion, such as in work by Dwyer[11] and by Monaco and Stornelli.[12] However, it will be seen that only the line-of-sight kinematics, given by Eq. (1), need be accurately modeled for VSC design.

Singularity Avoidance

The singularity occurring if the relative orientation between initial and target attitudes corresponds to $\phi = \pi$ or $-\pi$ in Eq. (3) can be eliminated as follows: the inertial reference axes with respect to which all orientations are referred are to be rotated by one-quarter turn about the initial Euler axis $e(0)$. If the target attitude had been the original inertial reference frame, then the initial attitude in its new parameterization becomes $\xi(0) = e(0)$ or $-e(0)$, whereas the final attitude likewise becomes, respectively represented by $-e(0)$ or $e(0)$. All intermediate attitudes $\xi(t)$ are automatically kept nonsingular by the control law itself, since, as shown by Dwyer,[21,22] no singularity can be introduced by the torque synthesis transformation itself.

III. Variable-Structure Control

A surface or manifold in the state space represents static relationships among the different state variables describing the behavior of the system. These relationships are enforced on the dynamic description of the system so that the resulting reduced-order dynamics will contain desirable features. The task is usually accomplished by opportune drastic changes in the structure of the feedback controller that induce velocity vector fields directed toward the sliding manifold in its immediate vicinity: hence the name variable-structure control.[19]

A nonlinear dynamic system model, which captures the essential features of the spacecraft model [Eqs. (1) and (2)], is given in state-space form below:

$$\frac{dx_1}{dt} = F(x_1)x_2 \tag{7}$$

$$\frac{dx_2}{dt} = f(x_1, x_2) + G(x_1, x_2)\mu \tag{8}$$

Systems modeled by Eqs. (7) and (8) are referred to in the VSC literature as being in *regular form*, as discussed by Luk'yanov and Utkin[23]; x_i $(i = 1, 2)$ are locally smooth coordinate systems defined on open sets of R^n ($n = 3$ for rigid spacecraft); $F(x_1)$ is an invertible smooth $n \times n$ matrix; G is also required to be an invertible matrix; f is a smooth vector field. For rotational dynamics, as in Eqs. (1) and (2), one has $F^{-1}(\xi)$ given by Eq. (6), whereas $G^{-1} = I^0$ for control with external torques, or $G^{-1} = I^0 - I'$ for reaction wheel control. Likewise, $f = (I^0)^{-1}\{(I^0\omega) \times \omega\}$ or $f = (I^0 - I')^{-1}\{h(\xi) \times \omega\}$ in each case; μ is then the vector of control torques τ or $-\tau'$. The pair of vectors (x_1, x_2), here (ξ, ω), will be denoted by x. For flexible systems, G may depend on the deformation state, unless x_1 includes elastic deflections as well as attitude parameters, and x_2 includes deformation rates as well as angular velocities. Two kinds of control laws will be considered, one to enforce the desired evolution of the attitude variables, and the other for the rate variables, as discussed next.

Sliding Motions in Attitude Variables

It is true that exogenous disturbances can be handled by outer loop filters and regulators, and elastic deformation effects can be handled directly through insertion thereof into the torque equations, or indirectly by singular perturbation synthesis, as discussed by Dwyer.[11] Nevertheless, plant modeling errors directly affect the inverse dynamics. The ensuing tracking error is interpreted in VSC as a deviation of the system state (x_1, x_2), from the *sliding surface* S defined below:

$$S = \{x \mid s := x_2 - m(x_1) = 0\} \tag{9}$$

$$m(x_1) := F(x_1)^{-1} f_{1d}(x_1) \tag{10}$$

where f_{1d} determines the desired form of evolution of x_1.

The perturbed system is best studied when expressed in the *surface coordinates* x_1, s, yielding Eqs. (12) and (13) below, by insertion of the following representation for x_2 in terms of x_1 and s:

$$x_2 = s + m(x_1) = s + F(x_1)^{-1} f_{1d}(x_1) \tag{11}$$

The transformed equations of motion then take the form

$$\frac{d}{dt} x_1 = f_{1d}(x_1) + F(x_1)s \tag{12}$$

$$\frac{d}{dt} s = \tilde{f}(x_1, s) + \tilde{G}(x_1, s)\mu \tag{13}$$

where $\tilde{f}$ and $\tilde{G}$ are given by the following expressions:

$$\tilde{f}(x_1, s) = f[x_1, s + m(x_1)] - \left[\frac{\partial m}{\partial x_1}\right] F(x_1)[s + m(x_1)] \tag{14}$$

$$\tilde{G}(x_1, s) = G[x_1, s + m(x_1)] \tag{15}$$

Equivalent Sliding Mode Control

The ideal evolution of the system on the suface S satisfies the *ideal sliding mode conditions* given by

$$s = 0 \tag{16}$$

$$\frac{d}{dt} s = 0 \tag{17}$$

known as the *invariance conditions*.[16,17]

The reduced-order feedback control that would turn S into an invariant manifold for the system is called the *equivalent control*, denoted here by μ_{eq}. The equivalent control, given by Eq. (18), is found by application of the invariance conditions [Eqs. (16) and (17)] to Eq. (13):

$$\mu_{eq}(x_1) = -\tilde{G}(x_1, 0)^{-1} \tilde{f}(x_1, 0) \tag{18}$$

It should be noted that on the sliding surface the evolution of the kinematic variable x_1 is given by Eq. (19), which justifies the form of Eq. (10):

$$\frac{d}{dt} x_1 = f_{1d}(x_1) \tag{19}$$

It can be shown that the equivalent control is exactly the control obtained by feedback decoupling, such as given in Refs. 8–10 whenever the sliding surface is defined by the corresponding commanded closed-loop trajectories.

Initial conditions might not lie on the ideal surface S. Moreover, if driven by the equivalent control, the system state will generally deviate from the sliding mode regime due to modeling errors in the computation of μ_{eq}. This motivates the VSC control strategies discussed next.

Additive VSC Correction

VSC is implemented by a choice of control law that counteracts deviations of the surface tracking error s in Eq. (9) from zero. Such a control law can be obtained by a choice of inputs satisfying the *sliding mode existence conditions*, which for a single input case are given by the following intuitively clear relations:

$$\lim_{s\to 0^+} \frac{d}{dt} s < 0 \tag{20}$$

$$\lim_{s\to 0^-} \frac{d}{dt} s > 0 \tag{21}$$

For general vector-valued situations conditions (20) and (21) are replaced by the Lyapunov-type condition below:

$$\frac{d}{dt} \|s\|^2 = 2s^T \frac{ds}{dt} < 0 \tag{22}$$

Condition (22) is verified by s when a control law of the following form is used in Eqs. (12) and (13):

$$\mu = \mu(x_1, s) = -\tilde{G}^{-1}(x_1, s)[\tilde{f}(x_1, s) + K \operatorname{sign}(s)] \tag{23}$$

Here sign(s) is the vector sign function

$$\operatorname{sign}(s) = [\operatorname{sign}(s_1), \operatorname{sign}(s_2), \ldots]^T \tag{24}$$

with

$$\operatorname{sign}(s_i) = \begin{cases} 1 & \text{if } s_i > 0 \\ -1 & \text{if } s_i < 0 \end{cases} \tag{25}$$

Note that $\tilde{f}$ is as in Eq. (14), $\tilde{G}$ as in Eq. (15), and K is any positive definite diagonal matrix of designer-selected weights: indeed, one then gets

$$\frac{d}{dt} \|s\|^2 = -2s^T K \operatorname{sign}(s) \tag{26}$$

The VSC law $\mu(x_1, s)$ is a correction to the equivalent control, to account for errors in its computation. Indeed, by setting s approximately zero near the switching surface S, one finds, by inspection of Eqs. (18) and (23),

$$\mu(x_1, 0) = \mu_{eq}(x_1) - \tilde{G}^{-1}(x_1, 0)K \operatorname{sign}(s) \tag{27}$$

The VSC gain K is experimentally (or optimally)[20] set sufficiently high to *guarantee the overshooting* of the ideal surface S, thereby triggering the switching logic. Only an estimate of the ideal control $\mu_{eq}(x_1)$ is therefore needed at each instant, although more accurate values of $\mu_{eq}(x_1)$ require less control effort.

Multiplicative VSC Correction

A VSC law can also be constructed as a *multiplicative correction*, rather than as an *additive correction*, to the equivalent control. To do this, it is first to be observed that control laws such as Eq. (27) are of a switching form, generically given below:

$$\mu_i(x_1, s) = \begin{cases} \mu_i^+(x_1) & \text{if } s_i > 0 \\ \mu_i^-(x_1) & \text{if } s_i < 0 \end{cases} \qquad i = 1, 2, \ldots, n \tag{28}$$

It is easily checked that the sliding mode existence condition (22), when applied to the system excited by the control law, Eq. (28), leads to the following characterization of the off-surface controls μ_i^+ and μ_i^- :[19]

$$\min\{\mu_i^-, \mu_i^+\} < \mu_{i,\,eq} < \max\{\mu_i^-, \mu_i^+\}, \quad i = 1,2, \ldots, n \tag{29}$$

A VSC law that can be easily shown to verify conditions (29) is as follows, with $k_i > 1$:

$$\mu_i(x_1, s) = -k_i \,|\, \mu_{i,eq}(x_1) \,|\, \operatorname{sign}(s), \qquad i = 1, 2, \ldots, n \tag{30}$$

Again *only an estimate* $\hat{\mu}_{i,eq}$ of $\mu_{i,eq}$ is needed in Eq. (30), since errors are detected by the switching logic, and the gains k_i can be set sufficiently larger than unity to guarantee reachability of the switching surface. Better accuracy in the estimation of $\mu_{i,eq}$ has the effect of requiring smaller VSC gains k_i and hence less control effort.

Chattering Avoidance

Undesirable chattering, evident in either VSC implementation above, can be avoided at the cost of tracking accuracy. This can be done by replacement of sliding surface reachability by *boundary-layer reachability*: that is, the system state (x_1, x_2) or (x_1, s) is required to be maintained only in a dead zone about the ideal surface, within designer-selected tolerances $\varepsilon_i > 0$ for each coordinate s_i.

The required *ε – accurate VSC* can be obtained by replacement, in Eq. (27) or (30), of the sign(s_i) function by the "ε-saturation" function defined below:

$$\operatorname{sat}_\varepsilon(s_i) := \begin{cases} \operatorname{sign}(s_i) & \text{if } |s_i| > \varepsilon \\ \varepsilon^{-1} s_i & \text{if } |s_i| \leq \varepsilon \end{cases} \qquad i = 1, 2, \ldots, n \tag{31}$$

The resulting controllers have been successfully employed with optimally varying gains in precision maneuvering of robotic manipulators under the name of *suction control,* in the work of Slotine.[20]

Sliding Motions in the Rate Variables

In the preceding considerations, the sliding surfaces represent relations between the pairs of state variables x_{1i} and x_{2i}, aimed at obtaining a reduced desirable evolution of the x_{1i} variables. It turns out that some other options may also be desirable in the controlled dynamic behavior of the state variables x_{2i}. In particular, detumbling or induced nutation[24] are also part of common requirements and objectives in spacecraft control problems. This poses the problem of synthesizing the appropriate surface equation on which to induce a sliding regime with equivalent dynamics which characterizes a desirable property of the x_2 state variables.

A desired reduced-order sliding dynamics may be specified as follows:

$$\frac{d}{dt} x_2 = f_{2d}(x_2) \tag{32}$$

The sliding surface $s = x_2 - m(x_1)$, on which the ideal sliding motion [Eq. (32)] is realized, is given by the solution of the following partial differential equation:

$$\frac{\partial m}{\partial x_1} F(x_1)m(x_1) - f_{2d}[m(x_1)] = 0 \tag{33}$$

The proof is based on the idea of having the ideal sliding dynamics for x_2 define the equivalent control function. By substituting $x_2 = m(x_1)$ in Eq. (32) and using Eq. (8) with $s = 0$, one finds

$$\frac{d}{dt}x_2 = \tilde{f}[x_1, m(x_1)] + \tilde{G}[x_1, m(x_1)]\mu_{eq} = f_{2d}[m(x_1)] \quad (34)$$

which yields

$$\mu_{eq}(x_1) = -G^{-1}[x_1, m(x_1)]\{-f_{2d}[m(x_1)] + f[x_1, m(x_1)]\} \quad (35)$$

On the other hand, the equivalent control is also given by Eq. (18), together with Eqs. (14) and (15) for $\tilde{f}$, $\tilde{G}$, respectively, at $s = 0$. Equating both expressions for the equivalent control from Eqs. (18) and (35), the partial differential equation [Eq. (33)] is obtained.

IV. VSC for Spacecraft Reorientation Maneuvers

In this section, the general results of Sec. III are applied to reorientation maneuvers. A *nonlinear sliding surface* is found which results in a *linear ideal sliding motion* for the kinematic variables. The ideal sliding motions, for each attitude parameter, can be prescribed as independent (decoupled) exponentially stable motions toward a desired final orientation. The solution of the single-axis problem provides a design avenue for the treatment of the multiaxial problem.

Single-Axis Reorientation

Single-axis maneuvers can be described in terms of the Cayley-Rodrigues kinematic description of the externally controlled spacecraft:

$$\frac{d}{dt}\xi = \tfrac{1}{2}(1 + \xi^2)\omega \quad (36)$$

$$\frac{d}{dt}\omega = \frac{1}{I^0}\tau^0 \quad (37)$$

or with I^0 replaced by $(I^0 - I')$ and τ^0 by $-\tau'$ for reaction wheel control. In either case, linear system equations may also be obtained by the alternative choice of the angular displacement as the orientation parameter [when the double integrator system $\dot{\theta} = \omega$; $\dot{\omega} = (1/I^0)\tau^0$ is obtained, or its counterpart with a reaction wheel]. However, Eq. (36) is used to stress the viability of applying the VSC approach directly to the nonlinear model, as well as to motivate the choice of control for the multivariable case. On the other hand, it can be shown that, by using the linear model and a linear sliding surface, not only is the globality of the sliding mode existence sacrificed, but one also loses the freedom of choice for the rate of convergence toward the desired rest orientation.

A desirable sliding surface is given below:

$$S = \{(\xi, \omega): s = \omega - 2[\lambda/(1 + \xi^2)](\xi - \xi_d) = 0,$$

$$\lambda < 0, \xi_d = \text{const}\} \quad (38)$$

where ξ_d is the desired final value of the orientation parameter. Its obvious advantage is that it provides a *linear* reduced-order ideal sliding motion, as can be easily seen from substitution of ω from Eq. (38), with $s = 0$, into Eq. (36):

$$\frac{d}{dt}(\xi) = \lambda(\xi - \xi_d), \qquad \lambda < 0 \quad (39)$$

i.e., $\xi = \xi_d$ is an asymptotically and exponentially stable equilibrium point. Using Eqs. (13–15) and $m(\xi) = 2\lambda(1 + \xi^2)^{-1}(\xi - \xi_d)$, one obtains, in this case,

$$\frac{ds}{dt} = \frac{[-\lambda(1 - \xi^2 + 2\xi\xi_d)]}{(1 + \xi^2)^2}s - \frac{2\lambda^2(\xi - \xi_d)(1 - \xi^2 + 2\xi\xi_d)}{(1 + \xi^2)^2} + \frac{\tau^0}{I^0} \quad (40)$$

The equivalent control is obtained either by using Eq. (18) or directly by enforcing the ideal sliding conditions [Eqs. (16) and (17)] on Eq. (40):

$$\tau^0_{eq} = 2I^0\lambda^2(1 + \xi^2)^{-2}(1 - \xi^2 + 2\xi\xi_d)(\xi - \xi_d) \quad (41)$$

Roughly speaking, this establishes the fact that faster maneuvers require larger applied controlled torques. The equivalent control constitutes a reference level for the computation of the actual VSC feedback gains. Using the reachability conditions (20) and (21), these gains can be synthesized by either additive or multiplicative VSC correction, as discussed in the previous section, as follows. The multiplicative option yields

$$\tau^0 = -k|\tau^0_{eq}| \operatorname{sign}(s), \qquad k > 1 \quad (42)$$

If a saturated controller is used, the sign function is simply replaced by the saturation function [Eq. (31)]:

$$\tau^0 = -k|\tau^0_{eq}| \operatorname{sat}_\varepsilon(s) \quad (43)$$

On the other hand, the additive correction approach requires setting $s = 0$ and $ds/dt = 0$ in Eq. (40). One then obtains an alternative expression for the controller:

$$\tau^0 = \tau^0_{eq} + r(\xi)s - k \operatorname{sign}(s), \qquad k > 0 \quad (44)$$

(or with sat instead of sign), with

$$r(\xi) = \lambda I^0(1 + \xi^2)^{-2}(1 - \xi^2 + 2\xi\xi_d) \quad (45)$$

This results in the stable surface dynamics $ds/dt = -k \operatorname{sign}(s)$, for which the reachability condition $s(ds/dt)$ $(= -k|s|) < 0$ is always satisfied. The control law given by Eq. (43) is preferred, however, for its inherent simplicity.

Multiaxial Reorientation

Multiaxial reorientation maneuvers are characterized by a desired orientation parameter vector $\xi(t_f) = \xi_d$ and a boundary condition of the form $\omega_i(t_f) = 0$, $i = 1, 2, 3$, where t_f denotes the final maneuvering time, and ω_i represents the angular velocity about the ith principal axis. In a VSC approach to this problem, one starts by considering ξ_d as an equilibrium point for the reduced-order ideal sliding motions taking place in the sliding surface $s = \omega - m(\xi) = 0$. By identifying ξ with x_1, using Eq. (10) and the nonsingularity of $F(\xi) = T(\xi)$, it follows that $m(\xi_d) = 0$. The ideal sliding motion is governed by the vector field $f_{1d}(\xi)$, which must have $\xi = \xi_d$ as an equilibrium point, i.e.: $f_{1d}(\xi_d) = 0$.

If a *linear* behavior of the sliding motion is preferred, then the desired vector field in Eq. (19) is of the following form:

$$f_{1d}(\xi) = \Lambda(\xi - \xi_d) \quad (46)$$

where Λ is an arbitrary constant *stable* matrix, thus inducing an asymptotically stable motion toward the desired orientation parameter vector.

By virtue of Eq. (9), the sliding surface is

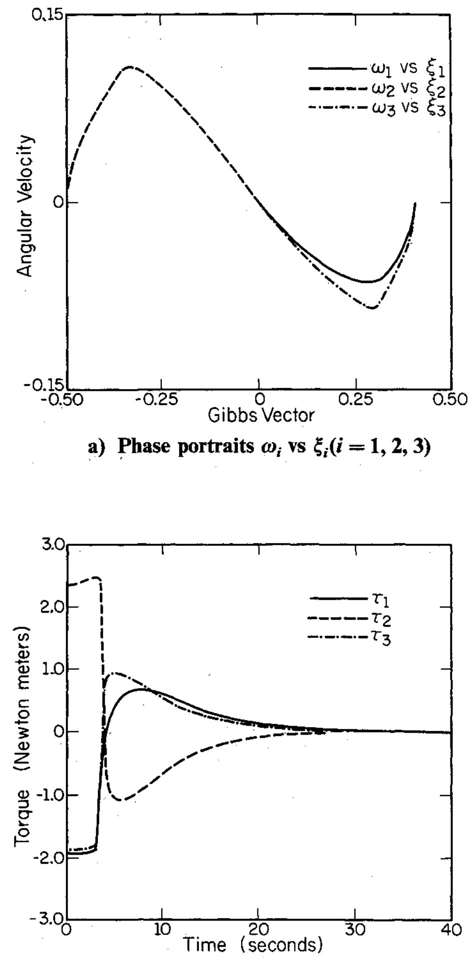
$$S = \{(\xi, \omega): s = \omega - F^{-1}(\xi)\Lambda(\xi - \xi_d) = 0\}$$

$$= \{(\xi, \omega): s = \omega - 2(1 + \xi^T\xi)^{-1}[I - \xi \times]\Lambda(\xi - \xi_d) = 0\} \quad (47)$$

since the surface equation (10) is now as follows:

$$m(\xi) = 2(1 + \xi^T\xi)^{-1}[\Lambda(\xi - \xi_d) - \xi \times \Lambda(\xi - \xi_d)] \quad (48)$$

If Λ is chosen to be *diagonal* ($\Lambda = \operatorname{diag}[\lambda_1, \lambda_2, \lambda_3]$, $\lambda_i < 0$ for all i), then the sliding motions are decoupled, and arbitrary time constants of exponential convergence can be individually



a) **Phase portraits ω_i vs $\xi_i (i = 1, 2, 3)$**

b) **Torques τ_i vs time $(i = 1, 2, 3)$**

Fig. 1 Multiaxial rest-to-rest maneuver.

imposed on the controlled kinematic motions toward the desired final orientation.

The equivalent control is obtained by using Eq. (48) in Eqs. (18), (14), and (15), with the appropriate interpretation of G, F, and f. Thus, for the externally controlled spacecraft, the equivalent torque is

$$\tau^0_{\text{eq}} = I^0 \frac{\partial}{\partial \xi}[F^{-1}(\xi)\Lambda(\xi - \xi_d)]\Lambda(\xi - \xi_d) - \{[I^0 F^{-1}(\xi)\Lambda(\xi - \xi_d)] \times [F^{-1}(\xi)\Lambda(\xi - \xi_d)]\} \tag{49}$$

whereas for control with orthogonal reaction wheels it is as follows:

$$\tau'_{\text{eq}} = [I^0 - I^a]\left\{\frac{\partial}{\partial \xi}[F^{-1}(\xi)\Lambda(\xi - \xi_d)]\Lambda(\xi - \xi_d) - h(\xi)[F^{-1}\Lambda(\xi - \xi_d)]\right\} \tag{50}$$

Evaluation of the gradients yields the same computed torques obtained by Dwyer.[11] A variable-structure controller can now be synthesized by any of the previous methods.

Alternatively, a crude estimate of the equivalent control can be obtained instead, as if the maneuver were of the single-axis type. This idea, although rather ad hoc, is in the same spirit as the method of the hierarchy of controls.[25] Thus, using the preceding expressions for the linearizing sliding surfaces, the following control laws are proposed for externally controlled spacecraft:

$$\tau^0_i = -k_i |\hat{\tau}_{i\,\text{eq}}| \operatorname{sign}(s_i), \qquad k_i > 1, \qquad i = 1, 2, 3 \tag{51}$$

with the equivalent torque estimates given below:

$$\hat{\tau}^0_{i\,\text{eq}} = 2I^0_i \lambda^2_i (1 + \xi^2_i)^{-2}(1 - \xi^2_i + 2\xi_i \xi_{id})(\xi_i - \xi_{id}), \quad i = 1, 2, 3 \tag{52}$$

Similar estimates hold for reaction wheel control torques. The ideal sliding motions of the controlled attitude parameters evolve according to the decoupled linear model given by Eqs. (9), (10), and (46) when $\Lambda = \text{diag}(\lambda_i)$.

To avoid high-frequency firing of thrusters and their associated chattering problem in the sliding dynamics, a saturated controller can be used as before:

$$\tau_i = -k_i |\hat{\tau}_{i\,\text{eq}}| \operatorname{sat}_\varepsilon(s_i) \tag{53}$$

This alternative is especially useful if reaction wheels or control moment gyros are used as actuators. The VS control given by Eq. (53) or (52) guarantees reachability of the sliding surface for any trajectory reasonably close to the intersection of the sliding submanifolds [to account for the approximate nature of Eq. (52)]. Example 1 illustrates the validity of this controller for the creation of sliding motions.

Example 1 (Multiaxial Rest-to-Rest Maneuver)

The simultaneous reorientation of all three body axes is considered here, for a spacecraft controlled with variable external torques, with the following inertia matrix taken from Vadali and Junkins[4] and Carrington and Junkins[6]:

$$I^0 = \text{diag}[114.562, 86.067, 87.212] \quad \text{kg-m}^2$$

In this example, a rest-to-rest maneuver is attempted using VSC for each axis, as if single-axis maneuvers were to be performed; i.e., the control law [Eq. (53)] is used for each axis. The target attitude parameter values are all zero. Each of the attitude parameters is ideally expected to evolve linearly, with independent time constants for each axis. In this example, the chosen exponential rates are $\lambda_1 = -0.15\ \text{s}^{-1}$, $\lambda_2 = -0.20\ \text{s}^{-1}$, and $\lambda_3 = -0.16\ \text{s}^{-1}$. Sufficient magnification gains for the equivalent control estimates were found to be given by $k_1 = 1.5$, $k_2 = 1.4$, and $k_3 = 1.7$. The value of ε in the saturated controller was taken as 0.001 rad-s^{-1}. Figure 1a depicts the different phase portraits of state variables corresponding to each axis. Figure 1b depicts the time evolution of the applied torques.

V. VSC of Detumbling Maneuvers

In this section, maneuvers seeking a null angular velocity about each of the body axes are considered, without regard for the final orientation parameters. It turns out that the VSC approach allows for various other kinds of angular velocity maneuvers, which are all conceptually treated in the same manner. For instance, despinning with respect to just one or two axes is possible while maintaining a constant angular velocity with respect to the remaining axes. For this reason, a constant vector of desired final angular velocities ω_d is considered here, without necessarily assuming its value to be zero. It is also possible to impose exponential rates of decay independently for the angular velocity profile components about each axis by prescribing appropriate nonlinear sliding surfaces, as will be demonstrated.

Detumbling with Undamped Ideal Dynamics

According to Eqs. (2) and (32), the detumbled motion in an externally controlled spacecraft is characterized by the desired angular velocity dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = 0 \tag{54}$$

i.e., the desired vector field of Eq. (32) is simply $f_{2d} = 0$. From Eqs. (2) and (4), the following equivalent torque vector is obtained:

$$\tau_{eq} = -(I^0\omega_d) \times \omega_d \tag{55}$$

i.e., once the desired angular velocity is reached, the average applied torque should be constant and dependent on the value of the desired angular velocity. In case $\omega_d = 0$, such an average torque is zero. (The same amount of torque is to be produced by the VSC in each direction for infinitesimal periods of time.) The solution of Eq. (33) is in this case given trivially by $\omega = m(\xi) = \omega_d$; i.e., the sliding surface is

$$S = \{(\xi, \omega): s = \omega - \omega_d = 0\} \tag{56}$$

For the additive VSC correction approach, one first considers the Lyapunov function $V(s) = s^T Ps$, with $P = I^0 = \text{diag}[I_1^0, I_2^0, I_3^0]$, on the surface differential equation obtained from Eqs. (2) and (56):

$$\frac{d}{dt}s = (I^0)^{-1}[I^0(s + \omega_d)] \times (s + \omega_d) + (I^0)^{-1}\tau^0 \tag{57}$$

to obtain the rate of change of $V[s(t)]$:

$$\frac{d}{dt}V(s) = 2s^T[\tau^0 - \tau_{eq}^0 + (I^0 s) \times \omega_d + (I^0\omega_d) \times s] \tag{58}$$

Therefore, the choice of control, give by

$$\tau^0 = \tau_{eq}^0 - (I^0 s) \times \omega_d - (I^0\omega_d) \times s - K\,\text{sign}(s) \tag{59}$$

with $K = \text{diag}[k_1, k_2, k_3]$, $k_i > 0$ for $i = 1, 2, 3$, guarantees a negative-definite time derivative of the Lyapunov function, and thus sliding surface reachability. If a saturated torque controller is preferred for the maneuver, then the controller is specified by replacement of sign by sat.

The options are again several: $\omega_d = 0$ implies total detumbling, whereas, say, $\omega_1 = \omega_2 = 0$, $\omega_3 = \omega_{3d}$ implies constant spin with respect to the third principal axis, such as for Earth pointing in orbit, in which case ω_{3d} is the orbit rate. In case $\omega_d = 0$, then $\tau_{eq} = 0$, so that the VS control law that achieves detumbling is decoupled and takes the following form, for the saturated torque controller case:

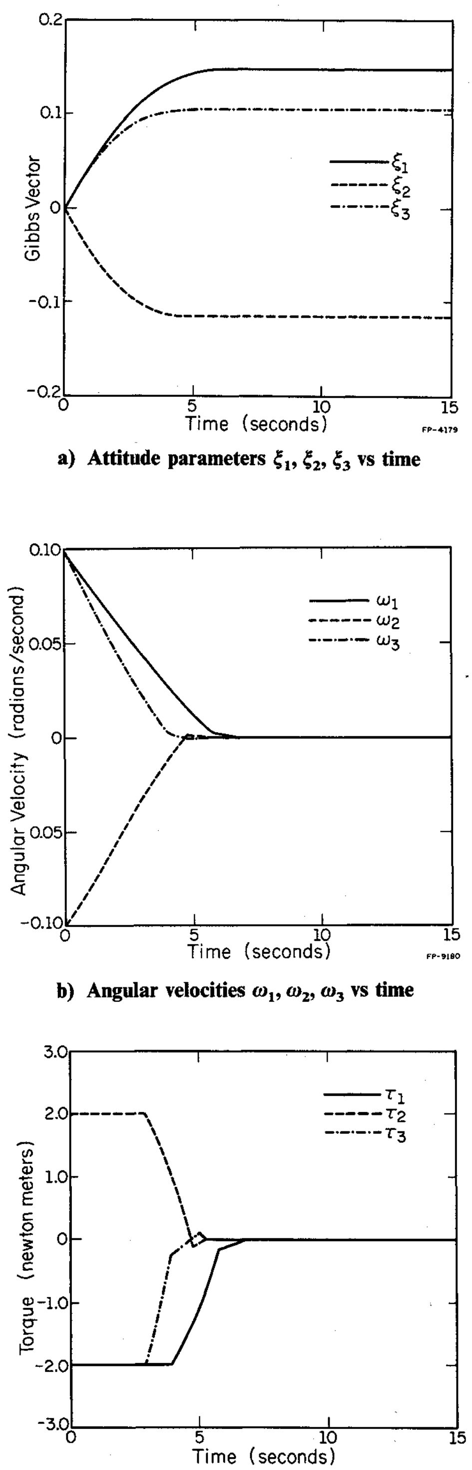
$$\tau_i^0 = -k_i I_i^{0\,-1}\,\text{sat}_{\varepsilon_i}(\omega_i) \tag{60}$$

Example 2 (Undamped Multiaxial Detumbling)

In this example, a detumbling maneuver is considered for a spacecraft with locked reaction wheels and externally applied torques. The matrix I^0 of spacecraft products of inertia is the same as in Example 1. The initial angular velocities are given by $\omega_1 = 0.1$, $\omega_2 = -0.1$, $\omega_3 = 0.1$, measured in radians per second. The initial orientation is assumed to be coincident with the inertial reference frame, i.e., $\xi_i(0) = 0$, $i = 1, 2, 3$. The saturated torque control law, for each control input, is given by Eq. (60), with $k_i = 2I_i^0$ chosen for $i = 1, 2, 3$, and the value of ε was assumed to be 0.002 rad-s^{-1}. The simulation results are shown in Figs. 2a–2c.

Detumbling with Exponentially Damped Ideal Dynamics

An ideal angular velocity sliding motion can also be considered to be represented by an exponentially stable decay toward rest. The partial differential equation (33) defining the sliding surface is easily solvable for the single-axis case. (In fact, it reduces to an ordinary differential equation.) In the multiple-axes case, the solution of the corresponding matrix partial differential equation will not be attempted here. However, the single-axis solution provides an attack scheme for the multiaxial detumbling problem, by considering single-axis sliding surfaces in combination with equivalent torque profiles derived



a) Attitude parameters ξ_1, ξ_2, ξ_3 vs time

b) Angular velocities ω_1, ω_2, ω_3 vs time

c) Applied torques τ_i vs time ($i = 1, 2, 3$)

Fig. 2 Undamped multiaxial detumbling.

from the ideal decoupled exponentially stable multiaxial dynamics. This results in an ideal stable dynamic motion dominated by linear exponential decay.

Damped Single-Axis Detumbling

Even though trivial per se, single axis detumbling will first be considered here, as essential motivation for the multiaxial case. The following ideal sliding dynamics for the evolution of the angular velocity is chosen:

$$\frac{d}{dt}\omega = \lambda\omega, \qquad \lambda < 0 \tag{61}$$

Letting the (unknown) sliding surface coordinate be defined by

$s = \omega - m(\xi)$, then the differential equation describing the evolution of this coordinate is

$$\frac{\mathrm{d}}{\mathrm{d}t} s = (I^0)^{-1}\tau^0 - \frac{1}{2}\frac{\partial m}{\partial \xi}(1+\xi^2)[s + m(\xi)] \tag{62}$$

The equivalent control is found by setting $s = 0$ and $(\mathrm{d}/\mathrm{d}t)s = 0$ in Eq. (62):

$$\tau^0_{\mathrm{eq}} = \frac{I^0}{2}\frac{\partial m}{\partial \xi}(1+\xi^2)m(\xi) \tag{63}$$

On the other hand, the ideal sliding motion given by Eq. (61) leads to a different expression for the equivalent control:

$$\tau^0_{\mathrm{eq}} = I^0 \lambda \omega = I^0 \lambda m(\xi) \tag{64}$$

Equating the expressions for the equivalent control and ignoring the trivial solution $m(\xi) = 0$, one obtains:

$$\frac{\partial m}{\partial \xi} = 2\lambda(1+\xi^2)^{-1} \tag{65}$$

Equation (65) has as a solution $m(\xi) = 2\lambda \tan^{-1}(\xi)$; i.e., the sliding surface is

$$S = \{(\xi, \omega): s = \omega - 2\lambda \tan^{-1}(\xi) = 0\} \tag{66}$$

The equivalent control thus takes the following form:

$$\tau^0_{\mathrm{eq}} = 2I^0\lambda^2 \tan^{-1}(\xi) \tag{67}$$

which, roughly speaking, indicates that faster stabilizing maneuvers require larger applied torques.

The existence of a sliding motion requires a variable-structure controller for which conditions (20) and (21) are satisfied. From Eqs. (62) and (63), one obtains

$$\lim_{s\to 0^+} \frac{\mathrm{d}}{\mathrm{d}t} s = (I^0)^{-1}\tau^{0+} - (I^0)^{-1}\tau^0_{\mathrm{eq}} < 0 \tag{68}$$

$$\lim_{s\to 0^-} \frac{\mathrm{d}}{\mathrm{d}t} s = (I^0)^{-1}\tau^{0-} - (I^0)^{-1}\tau^0_{\mathrm{eq}} > 0 \tag{69}$$

From here it is easy to conclude that the variable structure controller given by Eq. (30), but with s given as in Eq. (66), satisfies the existence conditions (68) and (69) while creating an asymptotically stable trajectory toward the origin. The saturated torque controller given by Eq. (43) can also be used, again with the new s.

Multiaxial Exponentially Damped Detumbling

A desired ideal sliding dynamics can be prescribed as a linear decoupled stable field vector: i.e., one lets the vector field f_{2d} of Eq. (32) (with $x_2 = \omega$) be as follows:

$$f_{2d}(\omega) = \Lambda(\omega - \omega_d) \tag{70}$$

where $\Lambda = \mathrm{diag}\,[\lambda_1, \lambda_2, \lambda_3]$ with $\lambda_i < 0$ for all i, and ω_d is a constant vector of desired final angular velocities, which is assumed here to be zero for simplicity.

The equivalent torque, shown below, is obtained in this case from Eq. (35), using Eqs. (2) and (70):

$$\tau^0_{\mathrm{eq}} = I^0[\Lambda\omega - (I^0)^{-1}(I^0\omega) \times \omega] \tag{71}$$

If the sliding surface $s = \omega - m(\xi) = 0$ is used, then the equivalent control is also expressible as follows:

$$\tau^0_{\mathrm{eq}} = -(I^0)\left[-\frac{\partial m}{\partial \xi} T(\xi)m(\xi) + (I^0)^{-1}(I^0\omega) \times \omega\right] \tag{72}$$

Similar formulas can be obtained for reaction wheel torques. The partial differential equation (33) can be obtained by comparing Eqs. (71) and (72). After substituting $\omega = m(\xi)$ and discarding the trivial solution case $\omega = m(\xi) = 0$, the following expression is obtained:

$$\frac{\partial m}{\partial \xi} = \Lambda T^{-1}(\xi) \tag{73}$$

with $T^{-1}(\xi)$ given by Eq. (6).

Rather than attempting to solve the preceding matrix PDE, a single-axis method will be presented which results in a locally stable ideal sliding motion with a simple controller structure. The approach is valid in the neighborhood of the origin of the phase space.

Individual sliding surfaces for each axis, suggested by Eq. (66), are used:

$$S_i = \{(\xi,\omega): s_i = \omega_i - 2\lambda_i \tan^{-1}(\xi_i) = 0, \quad \lambda_i < 0\}, \quad i = 1,2,3 \tag{74}$$

Using the invariance conditions (16) and (17), one immediately obtains, after differentiation of the surface coordinates, the equivalent control vector with the components

$$\tau^0_{1\,\mathrm{eq}}(\omega) = \lambda_1 I^0_1\left[\omega_1 + \frac{(\xi_1\xi_2 - \xi_3)\omega_2 + (\xi_1\xi_3 + \xi_2)\omega_3}{1+\xi_1^2}\right] - (I^0_2 - I^0_3)\omega_2\omega_3 \tag{75}$$

$$\tau^0_{2\,\mathrm{eq}}(\omega) = \lambda_2 I^0_2\left[\omega_2 + \frac{(\xi_2\xi_1 + \xi_3)\omega_1 + (\xi_2\xi_3 - \xi_1)\omega_3}{1+\xi_2^2}\right] - (I^0_3 - I^0_1)\omega_1\omega_3 \tag{76}$$

$$\tau^0_{3\,\mathrm{eq}}(\omega) = \lambda_3 I^0_3\left[\omega_3 + \frac{(\xi_3\xi_1 - \xi_2)\omega_1 + (\xi_3\xi_2 + \xi_1)\omega_2}{1+\xi_3^2}\right] - (I^0_1 - I^0_2)\omega_1\omega_2 \tag{77}$$

When substituted in the dynamic equations (2), the preceding torques yield the following ideal sliding dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_1 = \lambda_1\omega_1 + \lambda_1\left[\frac{(\xi_1\xi_2 - \xi_3)\omega_2 + (\xi_1\xi_3 + \xi_2)\omega_3}{1+\xi_1^2}\right] \tag{78}$$

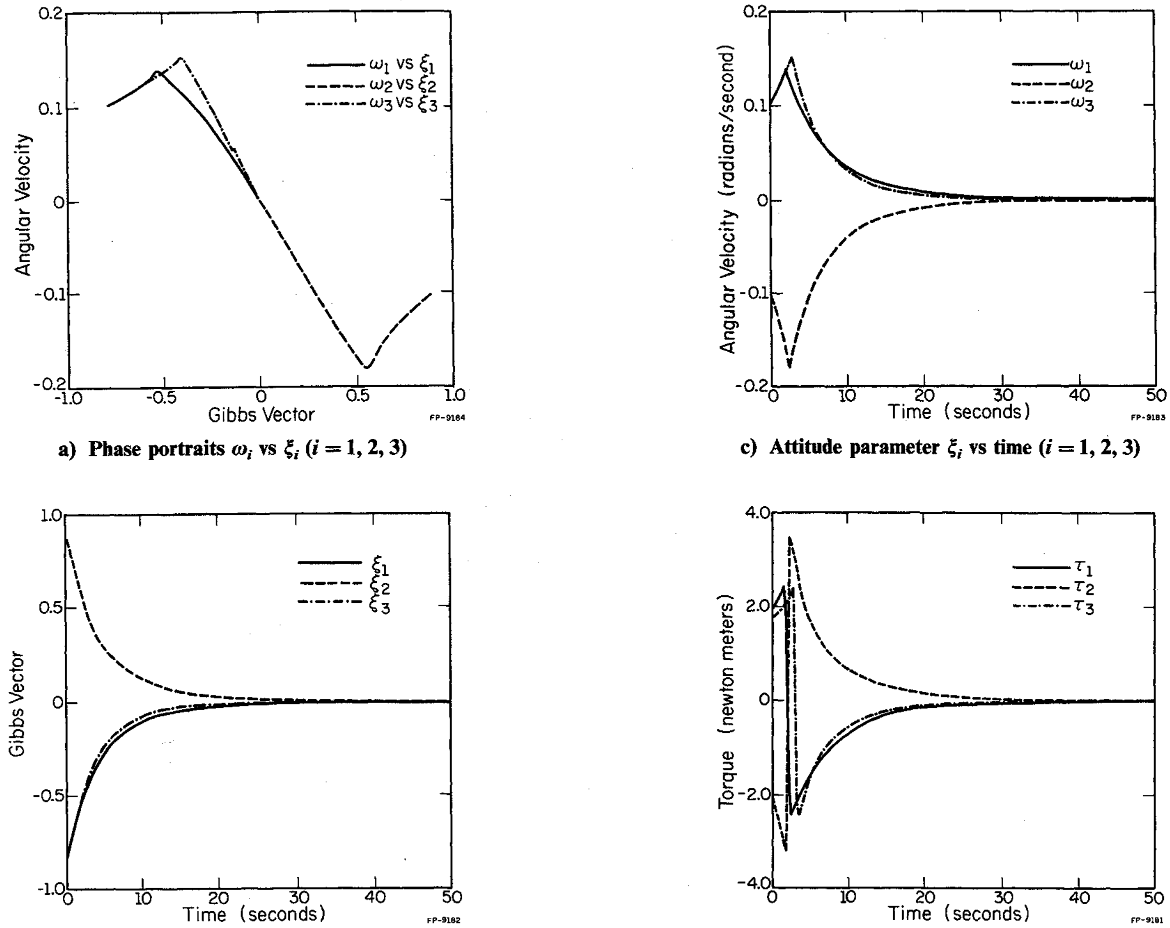
$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_2 = \lambda_2\omega_2 + \lambda_2\left[\frac{(\xi_2\xi_1 + \xi_3)\omega_1 + (\xi_2\xi_3 - \xi_1)\omega_3}{1+\xi_2^2}\right] \tag{79}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_3 = \lambda_3\omega_3 + \lambda_3\left[\frac{(\xi_3\xi_1 - \xi_2)\omega_1 + (\xi_2\xi_3 + \xi_1)\omega_2}{1+\xi_3^2}\right] \tag{80}$$

Since $\xi_i = \tan(\omega_i/2\lambda_i)$ for all i, it follows that for small values of ω_i, $i = 1, 2, 3$, the second terms on the right-hand side of Eqs. (78–80) are small, being at least of second-order magnitude in the angular velocity values. The closed-loop dynamics is thus dominantly linear and exponentially stable. Using the same argument, the torques given by Eqs. (75–77) are approximated by Eq. (71). This approximation is used as an estimated value of the equivalent torque vector, with components $\hat{\tau}^0_i$, in the synthesis of the VSC laws:

$$\hat{\tau}^0_i = -k_i|\hat{\tau}^0_{i,\mathrm{eq}}|\,\mathrm{sign}(s_i), \qquad k_i > 0, \qquad i = 1,2,3 \tag{81}$$

It is easy to show that in a sufficiently small neighborhood of the sliding surface $S = \cap S_i$, the controller given by Eq. (81), with s_i from Eq. (74), achieves sliding surface reachability. This is again done by verifying that $s_i(\mathrm{d}s_i/\mathrm{d}t) < 0$. If a saturated torque controller is preferred, the preceding sign function is replaced by the saturation function defined by Eq. (31). Similar results are obtained for reaction wheel torques, with $I^0_i\omega_i$ replaced by $h_i(\xi)$, and $\tau^0_{i,\mathrm{eq}}$ by $-\tau'_{i\mathrm{eq}}$.



a) **Phase portraits ω_i vs ξ_i ($i = 1, 2, 3$)**

c) **Attitude parameter ξ_i vs time ($i = 1, 2, 3$)**

b) **Angular velocity ω_i vs time ($i = 1, 2, 3$)**

d) **Applied torque τ_i vs time ($i = 1, 2, 3$)**

Fig. 3 Damped multiaxial detumbling.

Example 3 (Damped Multiaxial Detumbling)

The spacecraft parameters of Example 1 are again used in this example. Stabilized independent angular velocity profiles are obtained using the saturated torque controller corresponding to Eq. (81) with $k_i = 1.2$, $\varepsilon_i = 0.001$; the values of the exponential decays along each axis are taken to be $\lambda_1 = -0.14$, $\lambda = -0.16$, $\lambda_3 = -0.2$. Figures 3a–3d show the phase plane responses ω_i vs ξ_i, the individual angular velocity responses ω_i, and the torque profiles τ_i^0 for $i = 1, 2, 3$.

VI. Discussion

A general method of variable-structure control with nonlinear switching surfaces has been applied to multiaxial spacecraft reorientation and detumbling maneuvers. Cayley-Rodrigues attitude parameters were used to permit the rational, singularity-free, and unconstrained construction of exponentially decaying ideal motions for such attitude maneuvers. Under the resulting overshoot-and-correct control law, those spacecraft attitude parameters were shown to follow linear, decoupled, exponentially stable motions toward their target values, with independently chosen time constants. It was also shown that VSC-implemented multiaxial detumbling designed to imitate single-axis maneuvers results in nearly asymptotically exponentially stable controlled motions. More generally, any of several previously published attitude maneuvers, based on optimal control coupled with feedback linearization, can also be implemented but with greatly reduced on-line computational complexity, as well as improved robustness.

The emphasis in this paper has been on the ability of VSC implementation to permit such ideal-model following, with highly reduced command generation complexity. Robustness with respect to modeling errors, although not exhaustively examined here, is known to be inherent in VSC methods, as already shown in the referenced literature. In particular, the presence of structural deformations can be accounted for by higher VSC gains, thereby not affecting line-of-sight pointing accuracy, since the switching functions depend only on the kinematic part of the model.

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